3D vector fields

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Introduction

• Extension
  – Many methods of 2D vector fields can be extended to 3D, in particular as far as ODE and cell searching is concerned

• Main problem in 3D
  – Effective mapping to graphical primitives
  – Main aspects
    • Occlusion
    • Amount of (visual) data
    • Depth perception

Introduction

• Approaches to occlusion issue
  – Sparse representations
  – Animation
  – Color differences to distinguish separate objects
  – Continuity

• Reduction of visual data
  – Sparse representations
  – Clipping
  – Importance of semi-transparency
Introduction

• We are concerned with enhancements and special features for 3D vector fields
  – Glyphs
    • Like 2D, more special icons
  – Particle tracing
    • The same as in 2D
  – Time lines → time surfaces
  – P-space vs. C-space
    • Curvilinear grids – like 2D
  – LIC
    • Calculation is no problem, but rendering is difficult (display on 2D-screen, see next section)
  – Qualitative analysis at critical points

3D vector field topology
3D vector field topology

• Assumption
  – Critical point \( \mathbf{x}_0 \)
    \[ \mathbf{v}(\mathbf{x}_0) = 0 \quad \mathbf{J}_v(\mathbf{x}_0): \text{Jacobian matrix} \]

• Classification of critical points
  – In 3D, using the 3 eigenvalues of the Jacobian
  – There are 2 main cases
    • 3 real eigenvalues
    • 2 complex conjugate and 1 real eigenvalue

– Eigenvalues: each is real
  • \( \lambda_1 < \lambda_2 < \lambda_3 < 0 \) attracting node
  • \( \lambda_1 < \lambda_2 < 0 < \lambda_3 \) saddle node
  • \( \lambda_1 < 0 < \lambda_2 < \lambda_3 \) saddle node
  • \( 0 < \lambda_1 < \lambda_2 < \lambda_3 \) repelling node

– Eigenvalues: 1 real & 2 imaginary (like spiral in bathing tube)
  • \( \lambda_1 < 0 \): attracting relative to 0
    – \( \text{Re}(\lambda_2) = \text{Re}(\lambda_3) < 0 \) attracting focus
    – \( \text{Re}(\lambda_2) = \text{Re}(\lambda_3) > 0 \) repelling focus
    – Orientation of rotation depending on imaginary part
  • \( \lambda_1 > 0 \): repelling relative to 0
    – \( \text{Re}(\lambda_2) = \text{Re}(\lambda_3) < 0 \) attracting focus
    – \( \text{Re}(\lambda_2) = \text{Re}(\lambda_3) > 0 \) repelling focus
    – Orientation of rotation depending on imaginary part
3D vector field topology

### Critical points

- **Attracting and repelling point**
  - Flow perpendicular to surface
  - Tangential component of vector field → 0
  - Particle line start / end there

- **Field vortex (ger.: Wirbel)**
  - Motion of flow swirling rapidly around a center
  - Important since these are locations of loss of energy
  - Regions of concentrated vorticity (i.e. flow rotation)

- **Vorticity (ger.: Wirbelstärke or Vortizität)**
  - Central measure in fluid mechanics and meteorology
    - Measures tendency for elements of the fluid to "spin"
  - More formally
    - Related to amount of "rotation" (i.e. local angular rate of rotation) in a flow
    - (Pseudo-) vector field $\mathbf{\omega}$
      - defined as rotation of the velocity field $\mathbf{v}$:
      $$\mathbf{\omega} = \nabla \times \mathbf{v}$$
3D vector field topology

- **Examples for vortices**
  - Spiral motion with closed streamlines is vortex flow

- **Strategy for 3D vector field algorithms**
  - Find critical points
  - Classify eigenvalues of Jacobian $J_v(x)$
    - Attracting / repelling node
    - Attracting / repelling focus
    - Saddle, center
  - Combine critical points with particle lines
3D vector field topology

• **Separatrices**
  – Occlusion effects of stream surfaces
    ⇒ cluttered and hardly interpretable visualization

**Images:** Saddle Connectors – An approach to visualizing the topological skeleton of complex 3D vector fields, Theisel, Weinkauf, Hege, and Seidel

3D vector field topology

• **Separatrices** (cont.)
  – Instead use saddle connectors

**Images:** Saddle Connectors – An approach to visualizing the topological skeleton of complex 3D vector fields, Theisel, Weinkauf, Hege, and Seidel
3D vector field topology

- Example using saddle connectors
  - 13 critical points and 9 saddle connectors
  - LIC shows correspondence of skeleton and flow

3D vector field topology

- Stream line oriented topology of a 2D time-dependent vector field

LIC images at 3 different time slices
Tracking the locations of critical points as stream lines (red / blue / yellow)

Images: Saddle Connectors – An approach to visualizing the topological skeleton of complex 3D vector fields, Theisel, Weinkauf, Hege, and Seidel
Path lines
(Representation of particle lines)

• **Approach**
  – Randomly generate particles in the flow and trace the path of these particles over a specified time interval
  – Choose randomly
    • Starting positions
    • Starting time
    • Life time
  – Encode scalar values by
    • Color
    • Line width of path
    • Stream balls
    • Etc.
Path lines

• Including shading of lines
Path lines

- Glyphs along path line

[Diagram of car with path lines and glyphs along the lines]
Path lines

- Glyphs along path-line

Ribbons
(Representation of particle lines)
Ribbons

• **Approach**
  – Show rotation and divergence
  – Trace two close particles
    • Fill with polygon in between
    • Disadvantages
      – 2 traces
      – Difficult in case of separation (better keep the width constant)
  – Better: direct calculation of rotation
    • Map rotation of vector field directly on particle line
    • Vorticity
      – Measure for rotation of vector field
    • Streamwise vorticity
      – Projection of vorticity on vector of velocity

Ribbons

• **Decomposition of the Jacobian**

\[
J_x(\vec{v}) = \frac{1}{2} \left( J_x(\vec{v}) + (J_x(\vec{v}))^T \right) + \frac{1}{2} \left( J_x(\vec{v}) - (J_x(\vec{v}))^T \right)
\]

\[
= \Lambda + \Omega
\]

(deforation or stretching tensor)

(anti-symmetric
(local rotation)

– Furthermore

\[
J_x(\vec{v}) = \frac{1}{3} \text{spur}(J_x(\vec{v})) \cdot Id + \Sigma + \Omega
\]

• \(\frac{1}{3} \text{spur}(J_x(\vec{v})) \cdot Id\): can be interpreted as the expansion, i.e. local convergence or divergence of the flow field

• \(\Sigma\): symmetric matrix, representing the local shear
Ribbons

– Local rotation $\Omega$
  • This is a skew-symmetric matrix, called vorticity or spin matrix

  • Form of the rotation matrix
  $\Omega = \frac{1}{2} \begin{pmatrix}
  0 & -\omega_3 & \omega_2 \\
  \omega_3 & 0 & -\omega_1 \\
  -\omega_2 & \omega_1 & 0
  \end{pmatrix}$

  where the vector field $\vec{\omega} = (\omega_1 \quad \omega_2 \quad \omega_3)^T$

  is the rotation of the velocity, i.e. $\vec{\omega} = \nabla \times \vec{v} = \text{rot} \, \vec{v}$

Ribbons

– Angle of stream ribbon
  • Assuming Euler’s method (for simplicity only!) for integrating
  the orbit $x_n = x_{n-1} + \tau \vec{v}(x_{n-1})$

  • Let the band vector $b$ (determines direction of the ribbon), be a
  small vector perpendicular to the tangent vector of the orbit
    – Then $b$ is transformed according to
    $b_n = b_{n-1} + \tau \cdot J_x (\vec{v}(x_{n-1})) \cdot b_{n-1} + O\left(\|b_{n-1}\|^2\right)$

  • Neglecting higher order terms, expansion, and shear, it follows
    $b_n = b_{n-1} + \tau \cdot \Omega(x_{n-1}) \cdot b_{n-1}$

  • Local rotation perpendicular to the velocity direction
    – Project the new band vector $b$ after each integration step onto
      plane perpendicular to tangent vector of the orbit
    – Neglect rotation in path direction (already revealed by orbit itself)
Ribbons

• Divergence of vector field
  – Width of ribbon

• Advantage
  – "Curls" of the flow along the flow's direction can easily be recognized
Ribbons
Ribbons

- Aortic Blood Turbulence (by: R. Boon et al.)
  - Velocities and turbulence due to a pump in the aorta
  - Helps to understand how the pump affects blood flow near a patient's heart in order to better design the pump

Ribbons

- Alternative approach
  - Trace two close-by particles
  - Keep distance constant
Balls
(Representation of particle lines)

- Distance of balls
  - Velocity
  - Adaptive step size
- Radius of balls
  - Scalar value

- Disadvantage
  - Many triangles
Balls

Balls

Balls
Stream tubes
(Representation of particle lines)

Stream tubes

- **Polygonal object**
  - Spatial impression
- **Radius**
  - Scalar value
- **Calculation**
  - Circle → trace all elements
  - Trace line → then place circles and combine them
- **Advantage**
  - „Curls“ of the flow along the flow‘s direction can easily be recognized
Stream tubes

Stream tubes
Stream tubes

Density

0.0381 0.669 1.30 1.93 2.56

Stream tubes

Stream tetrahedra

• Combines advantages of balls and ribbons
  – Local rotation + velocity + divergence
    (rotation) (density) (size)
  – Very simple geometry
Stream tetrahedra

3D LIC
**3D LIC**

- **Calculation**
  - No problem – the same as in 2D

- **Representation**
  - Difficult interpretation
  - Requires volume visualization

- **Additional tools**
  - Clipping (plane, box, …)
  - Animation
  - Transparency
  - …

**3D LIC**

- **Missing continuity**
3D LIC

• Color differences to identify connected structures

3D LIC

• Animation
3D LIC

• Reduction of visual data
  – Restrict to volume of interest

3D LIC

• Reduction of visual data
  – Application of transparency
3D LIC

- Reduction of visual data
  - Deform clip object based on vector field

3D LIC

- Reduction of visual data
  - Clipping
  - Masking
3D LIC

- Reduction of visual data
  - Clipping

Combining different techniques
Combining different techniques

• **Examples**
  – Frequent combination of different techniques

• **Requirement of interactivity**
  – Efficient implementation
Combining different techniques

Visualization of blood flow